The Virtual Math Teams Project

An Overview of VMT

Introduction

The aim of the Virtual Math Teams (VMT) Project is to catalyze and nurture networks of people discussing mathematics online. It does this by providing chat rooms for small groups of K-12 students and others to meet on the Web to communicate about math. The vision is that people from all over the world will be able to converse with others at their convenience about mathematical topics of common interest and that they will gradually form a virtual community of math discourse.

For individuals who would enjoy doing math with other people but who do not have physical access to others who share this interest, the VMT service provides online, distant partners. For societies concerned about the low level of math understanding in the general population, the VMT service offers a way to increase engagement in math discourse.

The VMT Project was funded in Fall 2003 for five years by the US National Science Foundation. A collaboration of researchers at Drexel University and The Math Forum, the project is designing, deploying and studying a new online service at the Math Forum. This work is described below.

Background of the Math Forum

The Math Forum manages a website (http://mathforum.org) with over a million pages of resources related to mathematics for middle-school and high-school students, primarily algebra and geometry. This site is well established. A leading online resource for improving math learning, teaching and communication since 1992, the Math Forum is now visited by several million different visitors a month. A community has grown up around this site, including teachers, mathematicians, researchers, students and parents using the power of the Web to learn math and improve math education. The site offers a wealth of problems and puzzles; online mentoring; research; team problem solving; collaborations; and professional development. Studies of site usage show that students have fun and learn a lot; that educators share ideas and acquire new skills; and that participants become more engaged over time.

The Math Forum offers a number of online services, including the following. Most of these services were developed with research funding and volunteer support. Some of the established services now charge a nominal fee to defray part of their operating costs:

a. *The Problem of the Week*. This popular service posts a different problem every other week during the school year in a number of categories, such as math fundamentals, pre-algebra, algebra, geometry. Challenging non-standard math problems can be answered online or offline, with the opportunity for feedback from mentors on problem-solving and communication skills.

b. *Ask Dr. Math*. Students and others receive mathematics advice from professionals and expert volunteers.

c. *Math Tools*. Visitors to the site explore the world of interactive tools for understanding math concepts and communicate with teachers using them in their classrooms, discussing and rating the tools.

d. *Teacher2Teacher*. Classroom teachers and educators from around the world work together to address the challenges of teaching and learning math.

e. *Other*. Math Forum staff also provide online mentoring and teacher professional development, lead face-to-face workshops and work with teachers in their math classrooms.

f. *Virtual Math Teams*. The VMT service builds on the highly successful Problem-of-the-Week (PoW) service. Students who once worked by themselves on PoW problems can now work on more open-ended problems with a group of peers. This can be organized in a variety of ways and can bring many advantages as discussed in the following sections.

The VMT service design

The free VMT service currently consists of an introductory web portal within the Math Forum site (http://mathforum.org/vmt) and an interactive environment called VMT-Chat. VMT-Chat includes the VMT Lobby—where people can select chat rooms to enter (see Figure 1)—and a variety of math discussion chat rooms—that each include a text chat window, a shared drawing area and a number of related tools (see Figure 2).

Figure 1. The VMT Lobby.

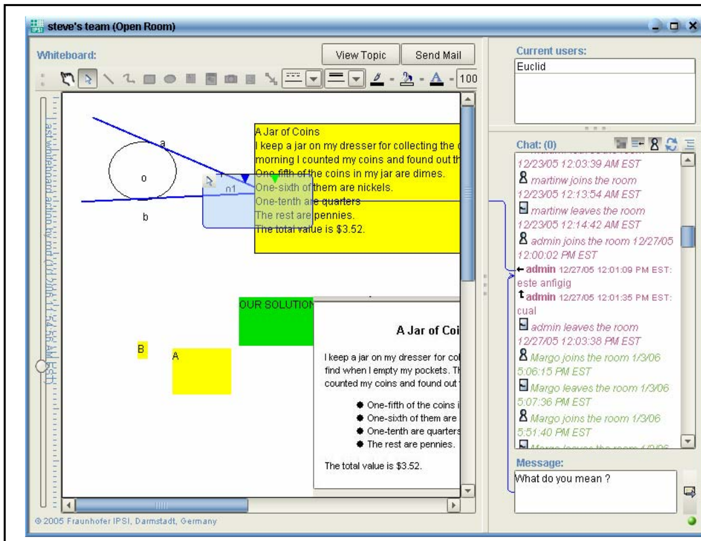

Figure 2. A VMT chat room.

Three types of rooms can be created in the lobby:

a. *Open rooms*. Anyone can enter these rooms and participate in the discussion – see Figure 1, where rooms are listed under math subjects and problem topics.

b. *Restricted rooms*. Only people invited by the person who created the room can enter.

c. *Limited rooms*. People who were not originally invited can ask the person who created the room for permission to join.

This variety allows rooms to be created to meet different situations. For instance, (a) someone can open a room available to the public; (b) a teacher can open a room for a group of her own students and choose who else to let in; (c) a person can just invite a group of friends.

Three general types of room topics are foreseen for VMT rooms:

a. *A math problem*. This could be a problem from the PoW service, or a similar challenging problem that may have a specific answer, although there may be multiple paths to that answer and a variety of explanations of how to think about it. In Figure 2, students discuss a problem about a jar of coins.

b. *A math world*. An open-ended math world describes a situation whose mathematical properties are to be explored creatively. The goal may be as much for students to develop interesting questions to pose as for them to work out answers or structural properties of the world.

c. *Open topic*. These rooms are open for discussion of anything related to math, such as perplexing questions or homework confusions.

Such flexibility allows the VMT service to be used in a wide range of ways and in limitless combinations and sequences:

1. For instance, teams of students from the same classroom might first use the VMT environment to work together on a series of PoW problems during class time, allowing them to become familiar with the system and build collaboration skills in a familiar social setting.
2. Later they could split up and join groups with students from other schools to explore more open-ended mathematical situations.
3. As they become more advanced users, they can create their own rooms and invite friends or the public to discuss topics that they themselves propose.

Through such sequences, people become more active members of a math-discourse virtual community and help to grow that community.

A new form of math education

The VMT project explores the potential of the Internet to link learners with sources of knowledge around the world, including other learners, information on the Web and stimulating digital or computational resources. It offers opportunities for engrossing mathematical discussions that are rarely found in most schools. The traditional classroom that relies on one teacher, one textbook and one set of exercises to engage and train a room full of individual students over a long period of time can now be supplemented through small-group experiences of VMT chats, incorporating a variety of adaptable and personalizable interactions.

While a service like PoW or VMT may initially be used as a minor diversion within a classical school experience, it has the potential to become more. It can open new vistas for some students, providing a different view of what mathematics is about. By bringing learners together, it can challenge participants to understand other people's perspectives and to explain and defend their own ideas, stimulating important comprehension, collaboration and reflection skills.

As the VMT library grows in the future, it can guide groups of students into exciting realms of math that are outside traditional high school curriculum, but are accessible to people with basic skills. Such areas include: symbolic logic, probability, statistics, digital math, number theory, infinity, group theory, matrices, non-Euclidean geometries. Many math puzzles and games also build mathematical thinking and stimulate interest in exploring mathematical worlds.

Ultimately, whole curricula within mathematics could be structured in terms of sequences of VMT topics with associated learning resources. Students could form teams to explore these sequences, just as they now explore levels of game environments. Such a Problem-Based Learning (PBL) approach could cover both the breadth and depth of mathematical fields, just as PBL curricula currently provide students at numerous medical schools with their academic training in face-to-face collaborative teams. In varying degrees, students could pursue their own interests, learning styles, social modes and timing. Assessments of student progress could be built in to the computational environment, supplementing and supporting teacher or mentor judgments. The collaborative, small-group VMT approach would be very different from previous automated tutoring systems that isolated individual learners, because VMT is built around the bringing together of groups of students to interact with one another.

Promoting knowledge building through math discourse

For most non-mathematicians, arithmetic provides their paradigm of math. Learning math, they assume, involves memorizing facts like multiplication tables and procedures like long division. But for mathematicians, arithmetic is not even part of mathematics. Math is a matter of defining new concepts and arguing about relations among them. Math is a centuries-long discourse, with a shared vocabulary, ways of symbolically representing ideas and procedures for defending claims. It is a discourse and a set of shared practices. Learning to talk about math objects, to appreciate arguments about them and to adopt the practices of mathematical reasoning constitute an education in math.

Classical training in school math—through drill in facts and procedures—is like learning Latin by memorizing vocabulary lists and conjugation tables: one can pass a test in the subject, but would have a hard time actually conversing with anyone in the language. To understand and appreciate the culture of mathematics, one has to live it and converse with others in it. Math learners have to understand and respond appropriately to mathematical statements by others and be able to critically review and constructively contribute to their proposals. The VMT project creates worlds and communities in which math can be lived and spoken.

Students learn math best if they can actively engage in discussing math. Explaining their thinking to each other, making their ideas visible, expressing math concepts, teaching peers and contributing proposals are important ways for students to develop deep understanding and real expertise. There are few opportunities for such student-initiated activities in most teacher-led classrooms. The VMT chat room provides a place for students to build knowledge about math issues together through intensive, involving discussion. Their entire discourse and graphical representations are persistent and visible for them to reflect on and share.

Evolving the VMT service design

The VMT service was not built from a fixed plan. It evolves.

The VMT project started by building on the success of the PoW service. In 2004, initial VMT sessions were held. Chat rooms were opened using a popular commercial chat system. Small groups of middle-school or high-school students were invited to work together in hour-long sessions on a PoW problem. An adult facilitator opened the room and announced the problem. If the students wanted to share a drawing or if they had technical problems, the facilitator assisted, but otherwise let the students work on their own. These early trials demonstrated that students were skilled at adapting to the chat environment and carrying on interesting mathematical discussions. However, it was clear that the software environment was too impoverished. It was hard to share drawings and to keep track of important ideas.

Later sessions experimented with introducing a shared whiteboard into the chat room. This allowed the participants to construct drawings related to the problem, to label the drawings and to post messages that remained visible on the board. This helped to overcome some of the technical difficulties. Unfortunately, it made the interactions more complicated. While some students invented effective group practices for taking advantage of the whiteboard, these were not universally used. It became clear that people needed time to get used to the environment and to learn effective procedures.

More recently, the software environment of chat with whiteboard has been supplemented with a number of additional tools or features designed to support math discourse and online interaction. Furthermore, attempts have been made to involve groups of students in sequences of consecutive sessions, in addition to one-shot events. The VMT Lobby was added to allow students to return to chat rooms or to locate sequences of rooms that teachers or VMT staff arranged for them

Perhaps most importantly, the nature of the problems offered has changed from the PoW format. As discussed above, different rooms have different kinds of topics. Some have individual problems, similar to the problems of the PoW service, but more oriented toward collaborative problem-solving. But other rooms have math worlds. These are open-ended situations that suggest situations, objects or patterns and relationships with interesting mathematical properties. In addition, students can open rooms for their own purposes. The nature of the topics and the ways they are presented strongly influence the nature of the interactions that take place in the rooms.

Supporting math discourse with software tools

Early theories of computer support for group work stressed the need to provide communication media, generally striving to duplicate as much as possible the features of face-to-face communication in situations where people were physically and/or temporally distant. Just as there are advantages (as well as disadvantages) of written communication over verbal, so there are advantages of particular computer-based media over face-to-face. The persistence of the written word in email, chat or threaded discussion is one important factor. In addition to supporting generic communication, it is possible for software environments to support group coordination and math problem-solving more specifically.

For instance, the addition of the shared whiteboard to the VMT environment not only facilitated the communication of graphical representations of mathematical situations (like geometry problems), but also allowed for the posting of text messages, equations and summary statements in small text boxes that remained on-screen while chat postings scrolled away. Students could decide to draw in different colors to coordinate simultaneous sketching. It would also be possible to add math symbols, labels for drawings or a simple calculator to help express and compute mathematical relationships.

An important tool in VMT-Chat provides the ability to reference from one text posting to a previous one or to a drawing area. This is an example of support for coordination. It helps in chats with several participants because when everyone is typing at once it is hard to tell which previous posting a new one is responding to. Furthermore, the referencing of an area of the whiteboard can support the mathematical work of defining specific areas in a drawing as corresponding to certain math objects.

For the development of the software environment, we began an intensive collaboration with researchers and developers at Fraunhofer Institute-IPSI in Darmstadt, Germany. They had developed a chat system with a shared drawing area and a referencing tool that provided both a form of threading in the chat and an integration of the drawing area with the chat. Their ConcertChat system formed the basis for VMT-Chat; it is licensed from IPSI for the VMT project. Working closely together, we not only improved the functionality of the chat rooms, but designed a Lobby for finding chat rooms.

It is possible to add many more software tools to VMT-Chat. The question is how to control the complexity of learning and using the system as it becomes more complicated. Separating the VMT-Lobby, help documents and archives of problems, resources and sample solutions from the chat rooms is one way to keep each part relatively simple.

Social practices that emerge in VMT

Perhaps more important than the design of the technological environment is the establishment of social practices to structure the behavior of participants in the chat rooms. Although this has been largely left up to the students in order to let them make VMT their own world, the VMT Project staff has tried to define expectations about how the space will be used. For instance, the ways in which students are invited to participate in VMT, the decor of the environment and the wordings of the room topics encourage an emphasis on math discourse.

Students enter the VMT environment with their previous experiences and bring along practices they have adopted in their school classrooms and social experiences. They are accustomed to tacitly agreeing upon ways of interacting. They are used to greeting people, starting a conversation topic, proposing new ideas, posing questions, taking turns, asserting themselves, saving face, correcting mistakes by themselves and others, coming to agreements and ending discussions. In VMT, this is all done through posting text in the chat stream and drawing on the whiteboard. It is normally done with strangers who are not visible. The VMT-Chat environment imposes a set of constraints and opportunities. It has aspects of a math classroom, a video game and an instant messaging exchange, as well as having unique characteristics. Groups of students adapt their familiar social practices to the peculiarities of the VMT-Chat environment. They spontaneously adopt and share methods of interaction – without necessarily being aware of them or able to explicitly describe them.

As researchers, the VMT staff tries hard to analyze the methods that groups use in VMT sessions. While these are in many ways unique to specific groups and sessions, one can also see patterns to the methods and structures to the sessions. Sessions typically start with mutual greetings and socializing. New users of the software spend some time experimenting with the tools or being trained in them. Eventually, someone suggests starting on the math topic and the question of how to begin arises. Math discussion often proceeds through sequences of math proposals, which themselves tend to have a typical structure of group interaction.

Analysis of group methods used in the VMT-Chat environment provides ideas for how to improve the software and the service design. It highlights where students have trouble making progress and where significant learning seems to be taking place.

Mentoring through guiding feedback

A major issue in the design of the VMT service is how to guide the student discourse so that it will build mathematical knowledge related to the given topic. In a traditional classroom, a teacher is present to impose structure, provide informational resources, direct the flow of ideas, evaluate proposals and assess learning. In a Problem-Based-Learning collaborative group, there is a professional mentor present to actively model methods of interaction and argumentation. In the long run for the VMT service, however, it is generally not possible to have an adult facilitator present. The design of the service must itself make up for this lack.

The Math Forum context sets the general tone that mathematics is the central concern. The way that a given chat room topic or math problem is written is designed to establish a certain attitude, expectation and perspective for the discourse to follow. In addition, the VMT experience is designed to encourage democratic discussion, where people know they will be listened to and supported; therefore they feel free to express themselves. Students may develop positive identities as people who enjoy math in situations where math is not a competitive performance that makes some feel stupid and others odd.

As the VMT service has evolved, it has become increasingly important to provide feedback to the students and to encourage them to come back repeatedly. While mentoring cannot be done during most VMT sessions, groups are encouraged to post summaries of their work and to request asynchronous feedback. Sometimes we provide a wiki for students to share their discoveries with other groups working on the same topics. VMT staff can go to a chat room the next day, review what took place, enter some feedback, guidance or suggestions and send the students an email encouraging them to come back to the room to read the feedback and perhaps hold a follow-up group session.

Building a community of math discourse

Ultimately, if students and teachers start to frequent the VMT service, share their group results, engage in multiple sessions and perhaps participate in other activities, they will start to form a user community. Teachers can interact at the site about the design of their favorite VMT math problems and share ways they have integrated VMT into their classrooms. Students can start to know each other from collaborating in groups together. They can participate in sequences of topics that build on each other. They can improve their collaboration and problem-solving skills and then start to mentor newcomers to VMT. As they become experienced with VMT, students and teachers can recommend improvements to the service and suggest variations to the topics.

We live in a society that is totally dependent upon knowledge of mathematics, but that does not value mathematical discourse outside of narrow academic or professional contexts. The Math Forum has gradually built an online realm in which a community of math discourse can be found. By virtue of its collaborative focus, the VMT service may be able to help that community prosper.

We are considering related services to help build a collaborative user community. An archive of student discoveries is one possibility that we are exploring using wiki technology, so that students can grow their own repository of discoveries. A teachers curriculum assistant site is another idea for supporting collaboration among teachers, who may want to know what topics worked for other teachers and share ways of involving students in math discourse. We would like to make the resources of the Math Forum digital library available to VMT participants in a relevant and useful way. And, of course, we are developing training materials (like this document) for students, teachers and researchers to introduce and explain VMT.

Studying group cognition

The VMT Project has the practical goal of establishing a new service at the Math Forum. It approaches this goal through a design-based research effort that starts simply and develops the design of the service through an iterative process of evaluating the results of trying new features. From a basic research perspective, this is a valid way to explore the nature of collaborative learning and small-group interaction in math chats.

More particularly, the VMT Project generates data illustrating *group cognition*. As virtual teams produce sequences of problem-solving moves, the actions of different participants merge into an integrated discourse. Cognition then emerges as an achievement of the group as a whole.

The VMT Project was designed as an experimental test-bed that captures lasting traces of collaborative interactions. The chat logs or persistent chat rooms preserve a rather complete record of the collaborative interactions that take place. The interactions involve challenging, creative problem-solving of mathematics, including critical reflection on the problem-solving and discourse about it. Thereby, the interactions produce numerous examples of group cognition in which teams produce cognitive results that cannot be attributed to any one individual but that arose out of the interactions among multiple participants situated in the group context. The records left for analysis by researchers contain most of the information that was available to the participants themselves. Since the students did not know each other from before the chat and could not observe each other except through the behavior that took place in the chat room, they could only understand each other's messages and actions based on what took place inside the chat room. The same information is available to researchers for understanding the messages and actions, providing an adequate record for analysis of how the group cognition took place. In contrast to classroom studies of face-to-face interaction, there is no need for videotaping and transcription, which introduce potential analytic difficulties.

The VMT Project allows researchers to see how small-group interaction and group cognition take place within a specific set of circumstances—e.g., small groups of K-12 students discussing math – with a particular form of technological mediation—i.e., chat with shared whiteboard and the features of VMT-Chat rooms. Synchronous math chats are different from forms of communication that have been studied more extensively, like asynchronous science threaded discussions or face-to-face social conversation. The VMT Project is able to study and document the distinctive nature of math chats and their specific potentials for fostering group cognition. In this way, it illustrates with one small example a much broader vision of engaged learning in online communities of the future.

Research in the VMT Project has been strongly influenced by visiting researchers. Jan-Willem Strijbos from the Netherlands led an intensive data coding approach. Analysis based on the coding was continued by Fatos Xhafa from Spain and Albania. On a more theoretical level, Stefan Trausan-Matu of Romania explored Bakhtin and related approaches to narrative interaction. Alan Zemel provided a strong conversation analysis perspective to understanding what takes place in the chats. Martin Wessner and Martin Mühlpfordt from Fraunhofer-IPSI in Germany each spent a summer in Philadelphia designing the VMT Lobby for group formation and expanding their ConcertChat software for VMT. Elizabeth Charles contributed a science-education perspective to the project from Canada. Currently, four doctoral students at Drexel University—Murat Cakir, Johann Sarmiento, Ramon Toledo and Nan Zhou (from Turkey, Columbia, Philippines and China)—are working on dissertations analyzing different aspects of the VMT Project and suggesting future developments for it. Information scientist Gerry Stahl is the Principal Investigator and Math Forum Director Stephen Weimar and anthropologist Wesley Shumar are co-PIs on the project.

A large number of publications have already emerged from the VMT project, including a book on group cognition, presenting the historical and theoretical background of the project, and many conference papers and journal articles (Stahl, 2006a, 2006b, 2006c, 2006d) (see http://www.mathforum.org/vmt/researchers/publications.html for a more complete list).

References

Stahl, G. (2006a). Analyzing and designing the group cognitive experience. *International Journal of Cooperative Information Systems (IJCIS)*. Retrieved from http://www.cis.drexel.edu/faculty/gerry/pub/ijcis.pdf.

Stahl, G. (2006b). Group cognition in an online chat community: Analyzing collaborative use of a cognitive tool. *Journal of Educational Computing Research (JECR) special issue on Cognitive tools for collaborative communities*. Retrieved from http://www.cis.drexel.edu/faculty/gerry/pub/jecr.pdf.

Stahl, G. (2006c). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. Retrieved from http://www.cis.drexel.edu/faculty/gerry/mit/.

Stahl, G. (2006d). Sustaining group cognition in a math chat environment. *Research and Practice in Technology Enhanced Learning (RPTEL), 1* (2). Retrieved from http://www.cis.drexel.edu/faculty/gerry/pub/rptel.pdf.